SOLUTION

CALCULUS II **QUIZ II** MATH-UA.0122-007

Write your solutions in steps.

1. (1 point) Your name:
2. (3 points) Compute: $\int \sin 2x \cos x \, dx$
3. (3 points) Compute $\int x\sqrt{1-x^4}\, dx$
4. (3 points) Compute $\int \frac{1}{x(x^2+1)^2}\, dx$

(2). $\int \sin 2x \cos x \, dx$

$= \int (2\sin x \cos x)\cos x \, dx$

$= -2\int \cos^2 x \, d\cos x$

$= -\frac{2}{3}\cos^3 x + C$

(3). $\int x\sqrt{1-x^4}\, dx$

$= \frac{1}{2}\int \sqrt{1-x^4}\, dx^2 \quad (u = x^2)$

$= \frac{1}{2}\int \sqrt{1-u^2}\, du \quad (u = \sin t)$

$= \frac{1}{2}\int \sqrt{1-\sin^2 t}\, d\sin t \quad 0 \le t \le \frac{\pi}{2}$. (NOTE $u = x^2 \ge 0$)

$= \frac{1}{2}\int \cos^2 t \, dt = \frac{1}{2}\int \frac{1+\cos 2t}{2}\, dt$

$= \frac{t}{4} + \frac{1}{8}\sin 2t + C$

$= \frac{t}{4} + \frac{1}{8}\cdot 2\sin t\cos t + C$

$= \frac{1}{4}(t + \sin t\cos t) + C$

$= \frac{1}{4}(\sin^{-1} u + u\cdot\sqrt{1-u^2}) + C$

$= \frac{1}{4}(\sin^{-1} x^2 + x^2\sqrt{1-x^4}) + C$

(4).

$$\frac{1}{x(x^2+1)^2} = \frac{A}{x} + \frac{Bx+C}{(x^2+1)} + \frac{Dx+E}{(x^2+1)^2}$$

$$= \frac{A(x^2+1)^2 + (Bx+C)x(x^2+1) + (Dx+E)\cdot x}{x(x^2+1)^2}$$

$$= \frac{(A+B)x^4 + Cx^3 + (2A+B+D)x^2 + (C+E)x + A}{x(x^2+1)^2}$$

So

$$\begin{cases} A+B=0 \\ C=0 \\ 2A+B+D=0 \\ C+E=0 \\ A=1 \end{cases} \Rightarrow \begin{cases} A=1 \\ B=-1 \\ C=0 \\ D=-1 \\ E=0 \end{cases}$$

$$\int \frac{1}{x(x^2+1)^2}dx = \int \frac{1}{x} - \frac{x}{x^2+1} - \frac{x}{(x^2+1)^2}\, dx = \ln|x| - \frac{1}{2}\int\left(\frac{1}{x^2+1} + \frac{1}{(x^2+1)^2}\right) d(x^2+1)$$

$$= \ln|x| - \frac{1}{2}\ln(x^2+1) + \frac{1}{2}\cdot\frac{1}{(x^2+1)} + C$$